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# Beyond the Mathematical Papyri: Functions and Contexts of Numbers in Ancient Egyptian Mathematics

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## Abstract

One of the most characteristic interpretations of ancient Egyptian mathematics, which has largely been maintained throughout its historiography since the first translations of the Rhind and Moscow papyri, is its eminently practical character; that is, empirical and concrete cases would constitute mathematical knowledge. Such anchoring in the concrete would be based on treatment of different mathematical themes through numerical cases. In the present investigation, we will propose a discussion of this position; for this, we consider, a type of research will be necessary that goes beyond the mathematical papyri and that, in a complementary way to the strictly mathematical approach, takes into account other dimensions of the field of Egyptology. Our main objective will be to propose the use of concrete numbers in Egyptian mathematical problems not as the exposition of a concrete case but as the mode of exposition of a paradigmatic case that makes it possible to understand the algorithm of the problem.


*Keywords:* Egyptian mathematics, Egyptian mathematical papyri, Rhind papyrus, arithmetic-calculative function, connotative-symbolic function.

## Introduction

The critical and interpretive analysis of the mathematical knowledge of ancient Egypt, based on the written remains that have survived from the Pharaonic past —mainly the Rhind Papyrus (pRhind) and the Moscow Papyrus (pMoscow), which, broadly speaking, can be dated to the Middle Kingdom (ca. 1980-1760 BC)[1]— is usually an area of knowledge traditionally located in the domains of influence of the history and philosophy of science, in addition to mathematical science. In this sense, it is usually expected that those who dedicate themselves to it are professional mathematicians or

[1] We use the chronology set out in Hornung, Krauss and Warburton 2006: 490-495.

philosophers whose field of research is the history of science. However, if we trace the origins of such analysis, we find that its first exponents were Egyptologists. Thus, for example, if we focus on the case of pRhind, the first investigations were related to the translation of the papyrus, and we find authors such as August Eisenlohr,[2] M. Léon Rodet,[3] Kurt Sethe,[4] and Thomas Eric Peet,[5] among others. The translation work of Peet, titled *The Rhind Mathematical Papyrus British Museum 10057 and 10058* (1923), has been, and in a certain way continues to be, a text that almost requires mention and citation. Following this, it is unsurprising that many of his opinions have been supported by a large part of later historiography. In particular, we can mention the following words:

> (…) while they were concerned mainly with practical problems, the Egyptians occasionally allowed themselves to observe and even record a result or a method that had no direct application to concrete facts of life. But there is no indication that such things were regarded as anything more than mere curiosities.[6]

Peet even went so far as to assert that Egyptian mathematics should not be judged from a presentist perspective.[7] However, according to later historiography, this assertion had no major consequences. Thus, for decades, one of the most characteristic interpretations of ancient Egyptian mathematics was its eminently practical character. In this sense, mathematical knowledge would be constituted by empirical and, moreover, concrete cases. Although it was often suggested in the first translations of pRhind that there was a kind of implicit algebra of equations in the Egyptian mathematical corpus —which would imply that scribes could work at a rather abstract level—[8] some critical voices were quick to point out their opposition. This is the case, to mention one of the authors of the earliest historiography, with M. Léon Rodet, who went so far as to assert that in pRhind there could only be purely arithmetic, and consequently concrete, procedures.[9] Such anchoring in the concrete would be based on the treatment of different mathematical themes through numerical cases.

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[2] Eisenlohr 1972 [1877].
[3] Rodet 1882.
[4] Sethe 1916.
[5] Peet 1970 [1923].
[6] Peet 1931: 438.
[7] See Peet 1934: 19-20.
[8] Vymazalová 2001: 578.
[9] See Rodet 1882: 5.

Therefore, to speak of Egyptian mathematics would be, in essence, to speak of arithmetic.

For at least two decades there have been lines of research that have attempted to rethink the image of Egyptian mathematics however in the most widely disseminated texts certain hermeneutics still survive that are embedded in the pristine reductionist views of Egyptian mathematics towards a concrete nature. An example of this can be found in the book *Einführung in die Ägyptologie: Stand — Methoden — Aufgaben* (1967) by Erik Hornung,[10] who briefly characterizes Egyptian mathematics in the following terms:

> The number of Egyptian mathematical treatises is less than that of medical treatises, although texts such as the Rhind Papyrus and the Moscow Mathematical Papyrus provide a good overview of the most common calculation operations. These are derived from experience and do not require any theoretical foundation; but the Egyptian could solve algebraic equations without complicated theories (…) The tasks in the textbooks were based on practical [understood here as concrete] exercises (…) This limitation is overcome by means of a rich technical vocabulary and some "actions of the imagination".[11]

In this research, we will propose a discussion of this position. To do so, we consider that a type of research will be necessary that goes beyond the mathematical papyri and that, in a complementary way to the strictly mathematical approach, takes into account other dimensions of the field of Egyptology. To achieve this, we will concentrate on one aspect in particular: the epistemic value of numbers and numerical cases within the problems of the Egyptian mathematical corpus.

In the case of Egyptian numeral forms, it is worth highlighting Kurt Sethe's work *Von Zahlen und Zahlworten bei den alten Ägyptern und was für andere Völker und Sprachen daraus zu lernen ist* (1916), which has the subtitle "A contribution to the history of arithmetic and language". Our first objective will be to analyze some contributions from this book, which we consider an important link for the understanding of the numeration of the country of the Nile both from a linguistic and purely mathematical point of view. Following the above, we will ask ourselves what types of functions and uses we can assign to numbers in certain non-strictly mathematical

[10] See Hornung 2000 [1967].
[11] Hornung 2000 [1967]: 119-120. My translation.

scenarios. In particular, we will inquire into the use of certain numerical quantities to express the elusive, or the extremely large, in a rather metaphorical sense in non-mathematical sources. This, through analogical reasoning, will lead us to a final objective, which will consist of proposing the use of concrete numbers in mathematical problems not as the exposition of an equally concrete case but as the mode of exposition of a paradigmatic case.

**Forms of numerical representation and characteristics of numeration**

We begin this section by making a necessary but brief allusion to something already quite well known: The Egyptian numeral system, at least in its hieroglyphic writing form.

The first comprehensive analysis of this topic can be found in the book—mentioned above in the introduction —titled *Von Zahlen und Zahlworten bei den alten Ägyptern*… by the renowned Egyptologist Kurt Sethe (1869-1934), which appeared a few years after August Eisenlohr's first translation of pRhind. This book was published in 1916 in Strasbourg, while the First World War was taking place. It is a truly comprehensive and fundamental study for the topic that concerns us in this article, in which the author's profound knowledge of the Egyptian language stands out, in addition to his constant references to the academic production that, at that time, had been developed on Egyptian mathematics by both Eisenlohr himself and Moritz Cantor (1829-1920). About the Egyptian numeral system—since Sethe also deals with other numerical subjects such as unit fractions—the book in question stands out for two reasons: firstly, its detailed analysis of its mathematical composition and the characteristics and mode of its writing; secondly, its linguistic-lexical investigation of the Egyptian words associated with the different numbers, in their relationship with Coptic and other Semitic languages, to elucidate the real meaning that such numbers had for the ancient inhabitants of the country of the Nile.

Let us now turn to the hieroglyphic numeral system. According to Kurt Sethe, its two most outstanding extra-mathematical characteristics are its remarkable antiquity—attested as early as predynastic times, in particular by the labels found in the U-j tomb at Abydos[12]—and its influence on the configuration of the Phoenician, Hittite and Minoan-Mycenaean[13] numerical systems. Sethe has characterized Egyptian hieroglyphic numeration in this way:

> The Egyptian number system, as we find it in use on monuments already at the beginning of [Egyptian] history (not later than 3300 BC), has a special numerical drawing for one and for each power from 10 to a million (…) It is therefore the same principle according to which the Phoenicians and in earlier times also the Greeks (in the so-called Herodian numerals) designed their numerical system, and which was essentially the basis of the Etruscan-Roman numerical system.[14]

According to the above quote, the hieroglyphic number system generally consists, of seven symbols and, as regards its numerical value, each one represents a certain power of 10. That is, each symbol is of the form $10^k$, with $k = 0, 1, 2, 3, 4, 5, 6$.

| Symbol | 𓏤 | 𓎆 | 𓍢 | 𓆼 | 𓂭 | 𓆐 | 𓁨 |
|---|---|---|---|---|---|---|---|
| Numerical value | 1<br>($10^0$) | 10<br>($10^1$) | 100<br>($10^2$) | 1,000<br>($10^3$) | 10,000<br>($10^4$) | 100,000<br>($10^5$) | 1,000,000<br>($10^6$) |
| Lexicon | *wꜥ* | *mḏ* | *št* | *ḫꜣ* | *ḏbꜥ* | *ḥfn* | *ḥḥ* |
| Order of magnitude | Units | Tens | Hundreds | Thousands | Ten of thousands | Hundreds of thousands | Millions |

Table 1: Egyptian hieroglyphic numerical system

As we can see in Table 1, the hieroglyphic system is based on 10—due, as mentioned prior, to the fact that the numerical value of each symbol is a power of 10—and not positional. Following the taxonomy proposed by Stephen Chrisomalis in his book *Numerical Notation. A Comparative History* (2010), we can characterize the hieroglyphic numeral system as intra-exponential,[15] whose main characteristic is determining how the numerical signs are constituted and combined within each base power. Furthermore, this system is:

[12] For more details on this topic, see Dreyer 1998.
[13] This has been investigated further in Chrisomalis 2003; 2010: 35.
[14] Sethe 1916: 2. My translation.
[15] Chrisomalis 2010: 11.

(a) *Cumulative-additive:* It is cumulative because to write any quantity that is a multiple of one of the powers of the base, the symbol in question must be repeated as many times as necessary; then, all the symbols represented must be added. As we can see, each symbol of a power of ten can be repeated only up to nine times, since the tenth time corresponds to the next power of ten. For example:

$$68{,}257 = 6 \cdot 10{,}000 + 8 \cdot 1{,}000 + 2 \cdot 100 + 5 \cdot 10 + 7 \cdot 1$$
$$= 6 \cdot 10^4 + 8 \cdot 10^3 + 2 \cdot 10^2 + 5 \cdot 10^1 + 7 \cdot 10^0$$

= 𓂭𓂭𓂭𓂭𓂭𓂭𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓆼𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺𓏺

In accordance with the above example, a first objection that has often been raised about the alleged limited mathematical ability of the ancient Egyptians is that the repetitive nature of so many signs could mean an obstacle to the representation of numbers and, therefore, to their cognition. Authors such as the Spanish Carlos Maza Gómez[16] have argued that the representational obstacle was solved in the development of hieratic writing, given that the links between the different hieroglyphic symbols produced that the hieratic numbering system—that is, the one used in administrative and, even more so, mathematical papyri—had a greater multiplicity of characters and, consequently, avoided falling into long series of repeated numerical symbols. Nevertheless, an alternative solution can be traced already in hieroglyphic writing itself, which leads us to the following characteristic.

(b) *Multiplicative-additive:* This form of writing for certain 'large' numbers was in use as early as the New Kingdom and is attested as late as the Ptolemaic period. It involves writing a multiplicative factor immediately below the hieroglyphic symbol for the powers $10^5$ and $10^4$, i.e. for 100,000 and 10,000.[17] For example, in a Ptolemaic inscription discovered by Karl Heinrich Brugsch,[18] the area of Egypt is expressed numerically, in terms of auras, using the symbol for 100,000 (𓆐) above the additive combination of symbols for the number 270 (𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓎆). While Brugsch reads this number as 100,270,

[16] See Maza Gómez 2009: 73.
[17] See Gardiner 1988 [1927]: § 259.
[18] See Brugsch 1968 [1883]: 604.

the more accurate guess would be 27,000,000, since 𓆐𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓎆 = 100,000 · 270 = 27,000,000. However, this multiplicative form of writing has rarely been used with the symbol for 1,000. An example of this usage is the Bilgai Stele, dating from the 19th Dynasty of the New Kingdom. According to Gardiner,[19] the following numerical expressions appear on lines 17 and 18, respectively:

- Line 17: 𓆼𓏺𓏺𓏺𓏺𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓏺𓏺 = 1,000 · 4 + 600 + 30 + 2 = 4,632.
- Line 18: 𓆼𓏺𓏺𓏺𓏺𓏺𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺𓏺𓏺 = 1,000 · 5 + 300 + 60 + 8 = 5,368.

**Functions of numbers in non-mathematical and mathematical contexts**

The conception and use of the notion of number as a mathematical entity that refers to some mathematizable reality has traditionally been analyzed by approaches to cognitive psychology, focusing on the interiority of the mind of ideal individuals abstracted from the contingencies of the context. However, this position has changed in recent years. An example of this approximate transformation is the article *The Cultural Challenge in Mathematical Cognition*, which claims the value of culture as a *conditio sine qua non* of the process of numerical cognition.[20] In the words of the authors:

> A strong influence of culture on mathematical cognition is (…) attested to by the extensive cultural diversity exhibited in which numerical tools are developed and used, how they are valued, taught, and culturally transmitted, and for which practical purposes they are regarded as relevant.[21]

Let us pause at the last part of the quote, the reference to ‘practical purposes’. Here we are told that number and its symbolic notation emerge and develop according to various purposes, and understanding these purposes is crucial to a full understanding of numeration as a cultural phenomenon. Furthermore:

[19] Gardiner 1912: tf. IV.
[20] Beller *et al.* 2018: 449. It is worth noting that this position recognizes as antecedents, among others, Núñez 2009.
[21] Beller *et al.* 2018: 449.

> It should be kept in mind that notational systems are not used for one purpose only, but are embedded in a rich set of cultural practices and may alternatively or exclusively be recruited for calculating a numerical value (as in arithmetic), for indicating a quantity (as in measures and prices), or for simply distinguishing entities (as in phone numbers or bus routes).[22]

In line with this positioning, we can therefore distinguish a series of functions of Egyptian numbers according to their various contexts of use. The first function is what we will call *denotative*. Here, the number establishes a biunivocal relationship with the number of empirical elements to which it refers. It is ultimately a typical counting function and constitutes, with all certainty, the most pristine type of use of numbers, judging by the inscriptions in historical sources. Let us look at some cases.

A group of labels from the aforementioned tomb U-j from Abydos contain only numerical symbols, presumably referring to the quantity of goods to which such labels were associated, such as the quantity of grain.[23] On the other hand, the mace-head of Narmer (Naqada III period) numerical symbols appear that, presumably, denote the spoils of war—or rather, tribute—after the victory over the Nile Delta (Fig. 1).[24] According to the inscriptions, the number of the spoils is estimated at around 400,000 oxen, 1,422,000 goats and 120,000 captives.

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[22] Beller *et al.* 2018: 452.
[23] See Dreyer 1998; Dreyer 2008; Cervelló Autuori 2016: 388-390; Imhausen 2016: 16, 22-23.
[24] See Imhausen 2016: 24-25. For a description of the mace-head and its scenes, see Quibell 1902: 39-41.

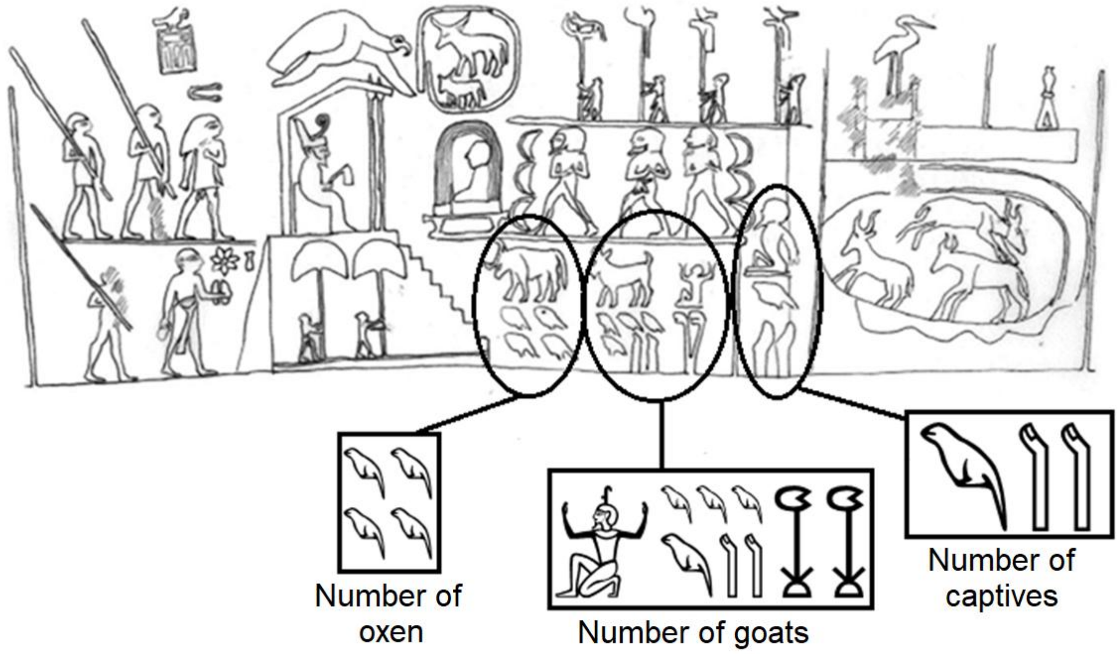
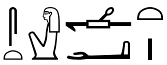
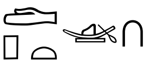
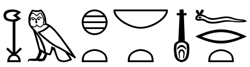
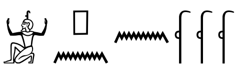
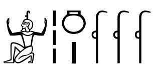



Figure 1: Denotative numerical function on the Narmer mace head (drawing by the author)

In expressions and phrases containing numerical quantities, these usually appear as adjectives or, as Sethe has pointed out for the million[25]—and, we add here, for other numerals as well—as a noun, both singular and plural. Consider the following examples:

-  (*st w*ʿ*t*): ‘a woman’: 1 as an adjective.
-  (*dp(w)t mḏw*): ‘ten ships’: 10 as an adjective.
-  (*ḫꜣ m ḫt nbt nfrt*): ‘a thousand of all good things’: 1,000 as a noun.
-  (*ḥḥ pt n rnpwt*): ‘this million years’[26]: 1,000,000 as a noun.
-  (*ḥḥw nw rnpwt*): ‘millions of years’[27]: 1,000,000 as a noun.

[25] Sethe 1916: 12-13.
[26] Sethe, *Urk.* IV, 306, 7.
[27] Sethe, *Urk.* IV, 358, 15.

The last expressions above bring up the fact that the quantities included in certain expressions are not always denotative but are, rather, a hyperbolic resource. In that sense, the number no longer indicates an exact quantification but, on the contrary, a deliberately large magnitude to connote something, roughly speaking, exaggerated and/or approximated according to some contextual objective. Therefore, we will call this second function as *connotative-symbolical*.

Kurt Sethe[28] has opportunely pointed out that *ḥfn* (100,000) and *ḥḥ* (1,000,000) possess, even intrinsically, an abstract significance of immensely countable and/or non-countable quantities. Thus, he emphasizes the fact that *ḥfn* acquires the sense of 'innumerable', while *ḥḥ* does so concerning 'infinite'. The case of the million is peculiar, at least semiotically, given the fact that the abstract idea of infinity—and eternity, if we refer to time—was embodied in the image of the god Heh. Following Elisa Castel,[29] said god can refer either to the member of the Hermopolitan Ogdoad—associated with the infinity of creation—or to the genius of air, eternity, and the 'millions of years of life' desirable for human existence. It is worth noting that the *ḥḥ* as a number disappeared during the Middle Kingdom, so in the hieratic papyri series of multiples of *ḥfn* were used to indicate numbers in millions. In monumental inscriptions, there remains a numerical vestige regarding the use of *ḥḥ* as an indefinite expression for large quantities,[30] which could therefore mean 'many' or 'a large number'.[31]

However, both *ḥḥ* and *ḥfn* often appear together in series of numerical symbols together with *ḏbꜥ*, *ḫꜣ* and sometimes *št*. In many of these series, from the New Kingdom and even reaching into the Ptolemaic period, the ring symbol  (*šn*) appears as an apparently disruptive element due to two reasons. The first of these is that it contradicts the general rule of writing hieroglyphic numerical symbols—that is, ordered from the highest to the lowest value—appearing both before and after *ḥḥ*. The second, and perhaps the most obvious, is that *šn* is not part of the set of symbols of the hieroglyphic numerical system already described. Therefore, how to interpret the presence of *šn* in a numerical context? A first answer was given by Sethe, and it constitutes a major

[28] Sethe 1916: 12-13.
[29] Castel 2001: 75. See Wilkinson 1992: 38-39; Kemp 2005: 117-118.
[30] Sethe 1916: 12.
[31] Faulkner 1991: 176; Erman and Grapow, *Wb* III, 152.

weak point of his book that we have already discussed: this Egyptologist argues that *šn* is nothing more than an addition without any defined meaning, and could be taken as 'years', 'one hundred thousand' or other similar expressions[32]. On the other hand, Batiscomb Gunn, in his review of Sethe's work, argued that perhaps, in order to lengthen the series of numerical symbols after *ḥḥ* and without having to give it a subordinate place, *šn* would have been assigned the value 1,000,000; consequently, *ḥḥ* would be equivalent to 10,000,000.[33] Furthermore, in texts somewhat outside the academic Egyptological field, the sign *šn* is often included as a fully-fledged integral part of the hieroglyphic numeration system. This is the case of Florian Cajori[34] who, in his monumental *A History of Mathematical Notations* (1928), gives the symbol Ω the value $10^7$ = 10,000,000.

In recent years, Luca Miatello, in his article *Expressing the Eternity as Seriality: On šn as a Number of Large Magnitude* (2016), has revisited this issue, offering a new, different, and more complete interpretation, to which we adhere. Indeed, he starts from the assertion that 'the Egyptians were clearly aware that the eternity and greatness of the universe shows itself in the infinitely large and the infinitely small',[35] with the infinitely large perfection and eternity being reflected in the series of powers of base 10.

Let us now consider the following scene from Theban tomb TT57 of Khaemkhat,[36] dating from the reign of Amenhotep III (Fig. 2). It shows Khaemkhat standing before the Pharaoh seated on his throne and holding a papyrus in his hands. Below a numerical expression containing three *šn* symbols can be seen, preceded by the word  (*dmḏ*) which can be translated as 'total'.[37] The transliteration of the expression containing the numerical expression is: *dmḏ šn 3 ḥḥ 3 ḥfn 3 ḏbꜥ 3 ḫꜣ 3 št 3*.

[32] Sethe 1916: 12, n. 6.
[33] Gunn 1916: 280.
[34] See Cajori 1993 [1928]: 12. Something similar occurs in Joseph 2001: 855, where the power $10^7$ is assigned not to *šn*, but to the solar symbol ⊙.
[35] Miatello 2016: 102.
[36] Porter and Moss, PM I[1], 115, (11). See Loret 1889: 129.
[37] Faulkner 1991: 267.

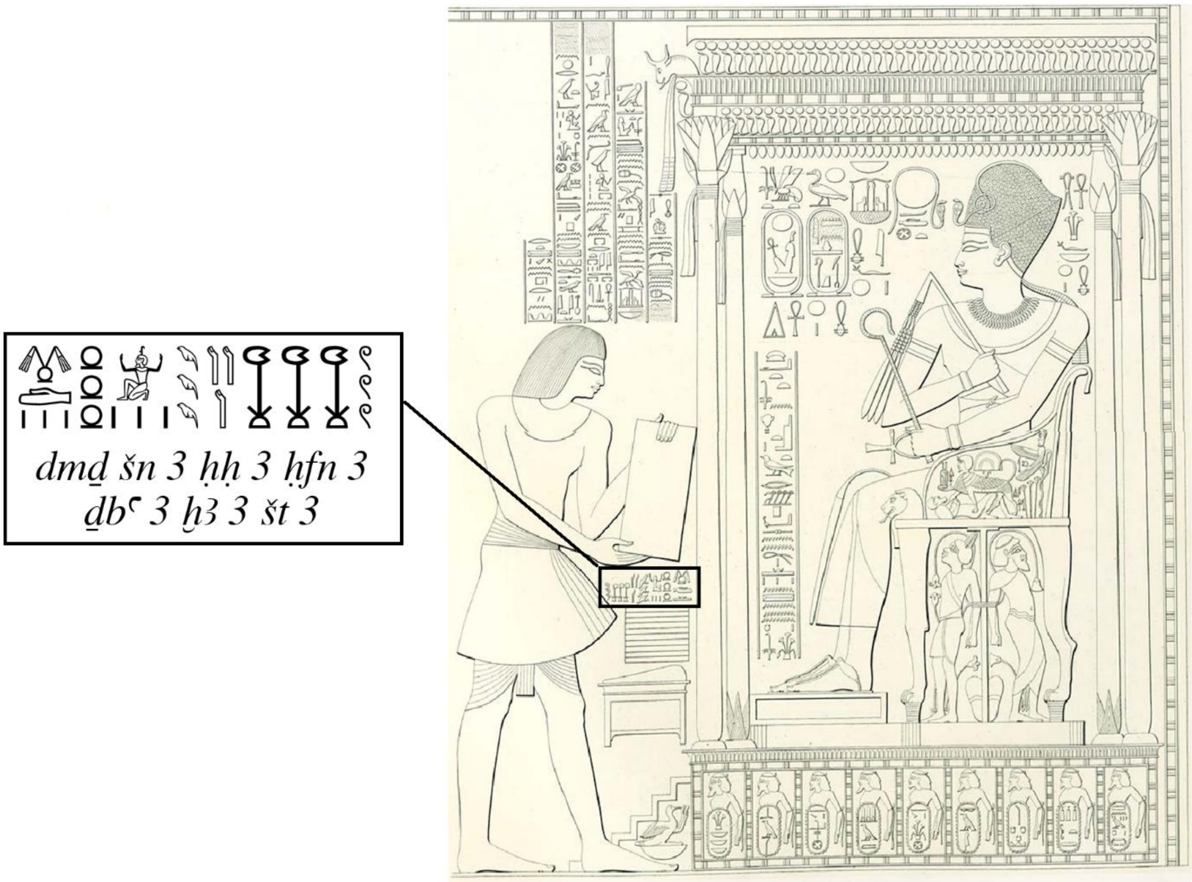


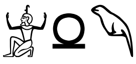

Figure 2: Numerical expression with *šn* in TT57 (adapted from Lepsius 1849-1859: Abth. III, Bl. 77c. Available on the website: https://edoc3.bibliothek.uni-halle.de/lepsius/tafelwa3.html)

The fact that *šn* is after *dmḏ* should clarify the fact that *šn* has a numerical value. So, if we consider that it is equal to 10,000,000, the inscription should be read as: 'total: 33,333,300'.

But what if *šn*, unlike the previous inscription, appears after *ḥḥ*? Miatello suggests that in that case, the numerical value differs, so that the arithmetic meaning of *šn* would be associated with some idea of positionality, based on its location with respect to *ḥḥ*. Indeed, considering the solar symbolism of 𓍶, in the sense of its association with the daily solar circuit, it was soon related to the abstract idea of 'eternity', apparently because the tied rope that formed the hieroglyphic symbol had no visible end.[38] Therefore, *šn* would be associated with the movement of the star Sotis expressed in days. To understand the importance of this interpretation, it must be remembered that

[38] Kemp 2005: 129.

the aforementioned star, belonging to the constellation of Canis Major, was the brightest in the Egyptian firmament and the Egyptian calendar  was based on its course.[39] Thus, 1 Sothic cycle is equal to 533,333.33 days, so 3 Sothic cycles are 1,600,000 days and 21 Sothic cycles are 11,200,000 days.[40] Taking this into account, Miatello proposes that the series formed by *ḥfn*, *ḥḥ* and *šn*, in its two possible appearances, should be taken as follows:

(a) *šn* to the right of *ḥḥ* equals the number half a million (500,000). Therefore:  = 1,000,000 + 500,000 + 100,000 = 1,600,000, which is the total number of days in 3 Sotic cycles.

(b) *šn* to the left of *ḥḥ* equals the number ten million (10,000,000). Therefore:  = 10,000,000 + 1,000,000 + 100,000 = 11,200,000, which is the total number of days in 21 Sotic cycles.

In short, in this interpretation we can point out that the symbol *šn* is part of hieroglyphic expressions that serve to indicate large astronomical numbers with an exaggerated or hyperbolic sense, suggesting an immensely large and ungraspable quantity. But such a quantity, in turn, is ordered to indicate the abstract idea of infinity and eternity. Thus, 'the expressed quantity alludes less to the figure itself than to the intention of alluding to infinity'.[41] In this way, we affirm that the numerical series analyzed are found in a symbolic and, even more, metaphorical relationship with the ideas of infinity and

[39] For more details, see Llagostera 2006-2007. According to Zulian 2004: 77, there are two phenomena associated with the star Sotis that can be considered relevant to the Egyptian calendar. The first of these is related to its period of visibility: although it was visible for a large part of the year, it stopped being visible for approximately 60 days and reappeared in the sky in mid-June, just before dawn, preceding the sun by a few minutes; this phenomenon is called the 'heliacal rising of Sotis'. However, the second phenomenon, which occurred approximately at the same time of year, corresponds to the beginning of the flooding of the Nile and, therefore, of the civil calendar. The latter lasts a total of 365 days (without leap years), while the duration of the time interval between two heliacal risings of Sotis (the 'Sothic calendar') is 365 ¼ days. Taking this into account, Miatello 2016: 108 considers a Sothic cycle as the conjunction between the civil and Sothic calendars, which occurs every 1,460 Sothic years or 1,461 calendar years.
[40] In this count of days, two numbers with great symbolic relevance in ancient Egypt are present: 3 and 7. See Bonanno 2016.
[41] Bonanno, 2016: 79. My translation.

eternity, since it arises under the need to represent ideas that in themselves are intangible and unrepresentable.

To conclude this section, we cannot fail to mention a third function of the use of numbers, perhaps the most obvious and immediate of all, which we will designate as *arithmetic-calculative*. This refers to the inclusion of specific numerical quantities in the different arithmetic operations.[42] In the mathematical papyri, traditional historiography has seen the Egyptian mathematical corpus as a set of arithmetic procedures referring to specific cases or elements, without any pretensions of generality. We oppose this interpretation. The following section, therefore, will be dedicated to developing our critical position.

**Epistemic relevance of numbers in Egyptian mathematical papyri**

We will begin this section by briefly referring to the development of the historiography of ancient Egyptian mathematics in recent decades. As mentioned in the introduction, it has often been considered a more or less disconnected set of practical and empirical problems, more suited to an architect or a surveyor than to someone we might consider a fully-fledged mathematician.

One of the leading historians who has challenged this view of the so-called 'problems' in the Egyptian mathematical papyri is Jim Ritter,[43] based on the precedent of Donald E. Knuth for the case of ancient Mesopotamian mathematics.[44] Based on Ritter's work, Annette Imhausen[45] proposed a real turning point in her doctoral thesis, *Ägyptische Algorithmen. Eine Untersuchung zu den mittelägyptischen mathematischen Aufgabentexten* (2003). According to her view, the Egyptian problems are not simple calculations but rather true algorithms. That is, they are a sequence of clearly defined instructions that, starting from initial numerical data, lead to a conclusion. Moreover,

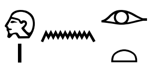
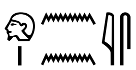
[42] For a detailed analysis of the algorithms of the different Egyptian arithmetic operations, see Reimer 2014.
[43] See Ritter 1995 [1989]; 1997.
[44] See Knuth 1972. His thesis was that Mesopotamian problems were not simple *ad hoc* calculations, but general procedures for solving all kinds of analogous situations. He therefore claimed that the problems were true 'algorithms'.
[45] See Imhausen 2003.

this interpretation ‘also allows for comparing the problem texts (*Aufgabentexte*) with each other and for establishing relationships between individual problems and groups of problems’.[46] The purpose of Imhausen's use of the category 'algorithm' is to distill the mathematical 'substance' that is contained in the rhetorical texts of the Egyptian papyri.[47] Given this, we might ask which elements have a role of epistemic precedence in algorithms: the steps and instructions, on the one hand, or the numerical data and calculations, on the other.

To begin to answer this question, we must bear in mind that in many of the problems in the mathematical papyri, the calculations are not always detailed. This led M. Caveing to assume that mental calculation was particularly developed in Egypt. Furthermore, he has argued that:

> The mental calculation, or unwritten auxiliary calculations, suggests that the written part consists of results organized for the purpose of communicating a teaching that contains only the essentials and shows only numbers carefully chosen for this purpose.[48]

The careful choice of numerical quantities for the initial data of the algorithm, for example, to facilitate calculations, is not a mere accidental fact but a mathematical intention, suggesting that the emphasis is placed on the following of a reasoning and not on a concrete accident. For example, if, according to pRhind 48, the area of a circle of diameter d is obtained by the operation $\left(1-\frac{1}{9}d\right)^2=\left(\frac{8}{9}d\right)^2$, it is not at all fortuitous that the value $d = 9$ is taken as the initial numerical data. According to this, the mathematical papyri would not be strictly a collection of problems, but rather a collection of procedural models that would transmit the teaching through example. This would explain the fact that many problems, at least those that have a heading in text form, begin with the expressions:

-  (*tp n ỉrt*): ‘method for making/calculating’.[49]
-  (*tp n nỉs*): ‘method for calculating’.[50]

[46] Imhausen 2003: 26. My translation.
[47] According to the conclusions contained in Imhausen 2002: 158.
[48] Caveing 1994: 381. My translation.
[49] pRhind 1, 39, 41, 50-52, 62, 65; pMoscow 2-25; pUC32162-1.
[50] pRhind 44, 56.

- (*tp n ḏbꜣ*): ‘method for replace/exchange’.[51]
- (*tp n psš*): ‘method of distribution’.[52]
- (*tp n ḥsb*): ‘method of calculating (accounts, taxes)’.[53]
- (*tp n ỉst*): ‘method to convert/recalculate’.[54]

Furthermore, we must not forget the words with which pRhind begins (Fig. 4):

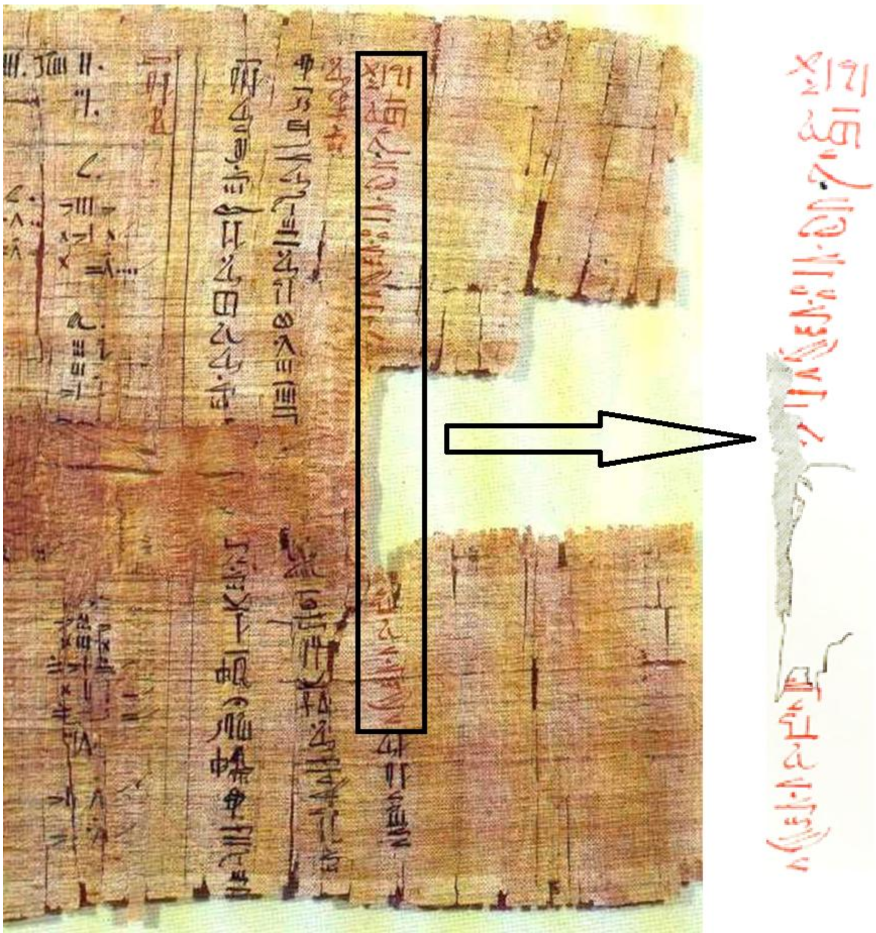
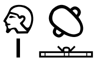


Figure 4: Partial photograph of the initial text of pRhind, usually called the 'prologue', pointing to the hieratic text transcribed in hieroglyphics, transliterated and translated below[1] (My intervention on photography). © The Trustees of the British Museum. Shared under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0) licence.

[51] pRhind 73, 77-78.
[52] pRhind 64.
[53] pRhind 67.
[54] pRhind 49.

*tp-ḥsb n hꜣt rẖ(t) ntt nbt snkt m ẖt (…) štꜣt nbt*

‘*Correct method* to enter into the knowledge of all things that exist (…) [of] all the secrets’.

Let us focus, for the moment, on the initial expression  (*tp-ḥsb*). According to the *Wörterbuch der ägyptischen Sprache*, the verb *ḥsb* means ‘to account’ (*rechnen*, *barechnen*) and the compound term *tp-ḥsb* takes on the meaning of ‘exact calculation’, ‘precision’ (*richtige Berechnung*, *Richtigkeit*).[55] Hannig’s dictionary, finally, confirms this interpretation.[56] According to this literal meaning, the initial expression of pRhind clearly shows the importance that the ancient Egyptian scribe-mathematician would have placed on the precision, correctness, and orderly progress of the steps in solving the different problems.

In his attempt to demonstrate that ancient Egyptians were capable of a certain degree of systematization and abstraction in thought, Théophile Obenga draws his attention to the term *tp-ḥsb*, which he translates as ‘correct method’ (*méthode correcte*).[57] With this translation, it would be possible to demonstrate that there was a crucial ‘theoretical’ need in the mathematical activity of the time of the pharaohs:

> The method is, in essence, a logical operation that aims at achieving a specific result, pointing out certain errors to avoid. In short, it is about reasoning correctly and avoiding errors. (…) Since ancient Egypt (…) the decisive philosophical problem, dealing with mathematics in particular and science in general, the central problem, therefore, has always been that of knowing the logical operations through which the human mind will reach the truth, avoiding error: it is what the Egyptians called *tep-heseb*, ‘rules’, ‘correct method’ to study nature in all its nooks and crannies in an exact manner.[58]

If we accept these words of Obenga, which are clearly in line with Imhausen's more recent position, we must finally ask ourselves: what functions do numbers fulfill in Egyptian mathematical algorithms? In the Egyptian mathematical method, numerical quantities serve as models to demonstrate the normative steps of resolution, since

[55] Erman and Grapow, *Wb* III, 16-ff.
[56] Hannig 1995: 603.
[57] See Obenga 1990.
[58] Obenga 1990: 361.

these are not exhausted in them. It is, in a certain way, a structuring of mathematical thought that provides knowledge from example.

Here we could draw a parallel with some of the functions described in the previous section:

- There is an unavoidable *arithmetic-calculative* function, present in those problems that do present the calculations properly written.
- The initial numerical data, carefully chosen, fulfill a *connotative-symbolic* function: rather than necessarily denoting a concrete empirical case, they connote quantities that establish a numerical precedent. This allows the algorithm, as an 'exemplary example', to be later applied to other similar cases.

Let us pause momentarily on the second of the functions just described. It indicates that there is no sharp separation between the particular and the general, but that the particular is capable of being generalized to more cases with similar conditions. Conversely, the general in the mind needs to be expressed through a particular form. Following Luigi Fraschini,[59] between the particular and the general there is an analogy of proportion. Let us explain this question as follows:[60] if we denote as $t_0$ = particular arable land, $s_0$ = concrete surface of that particular arable land, $t$ = arable lands (thought or real) with conditions analogous to the particular one, and consequently $s$ = surface of those thought or real lands, then for an Egyptian, it is true that $t_0$ is to $s_0$ as $t$ is to $s$ (in usual symbolic notation, $t_0 : s_0 :: t : s$). In short, the process of going from the concrete to the general occurs because both terms are analogous in proportion.

The appeal to the concrete to allude to the general, according to this analogy of proportion, is none other than the connotative-symbolic function of the number already defined. This approach has roots in the *indirect nature* of Egyptian thought, as R. A. Schwaller de Lubicz has been able to characterize it: 'a definite form will be used to evoke the idea of that form, that is to say [that] the abstract complex directs this definite form'.[61] Therefore, we can affirm that the fact of resorting to a definite or concrete form

[59] See Fraschini 2013.
[60] See Gerván 2024: 234.
[61] Schwaller de Lubicz 1985 [1963]: 111.

responds to the need to evoke an idea or concept *in mente*, and what defines its form is that idea, not the other way around.

## Final remarks

To conclude this work, we can see how the arithmetic-calculative and connotative-symbolic functions, defined here as belonging to the mathematical papyri, are not far removed from the functions of numbers analyzed for contexts that are not strictly mathematical. This reveals to us how, in order to appreciate the depth, richness and ubiquity of the Egyptian mathematical corpus, we must place it within the broader framework of Egyptian cultural practices. Only in this way will we be able to fully understand, at least in the Egyptian case, the fact that all mathematical cognition is inextricably linked to culture. Ultimately, and beyond the first tasks of translation carried out at the end of the 19th century and the beginning of the 20th century, this shows us that the study of the mathematics of the country of the Nile must, today more than ever, have full citizenship within the disciplinary field of Egyptology.

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